Physics Letters A 380 (2016) 3970–3976

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

A minimalist approach to conceptualization of time in quantum theory

Hitoshi Kitada [a], Jasmina Jeknić-Dugić [b], Momir Arsenijević [c], Miroljub Dugić [c,*]

[a] Department of Mathematics, Metasciences Academy, Japan
[b] Department of Physics, Faculty of Science and Mathematics, University of Niš, Serbia
[c] Department of Physics, Faculty of Science, University of Kragujevac, Serbia

ARTICLE INFO

Article history:
Received 20 July 2016
Received in revised form 7 October 2016
Accepted 10 October 2016
Available online 14 October 2016
Communicated by P.R. Holland

Keywords:
Foundations of quantum mechanics
Functional analytical methods
Lagrangian and Hamiltonian approach

ABSTRACT

Ever since Schrödinger, Time in quantum theory is postulated Newtonian for every reference frame. With the help of certain known mathematical results, we show that the concept of the so-called Local Time allows avoiding the postulate. In effect, time appears as neither fundamental nor universal on the quantum-mechanical level while being consistently attributable to every, at least approximately, closed quantum system as well as to every of its (conservative or not) subsystems.

1. Introduction

Schrödinger's Quantum Mechanics in [1–3], is timeless when he introduced his fundamental equation as a time-independent equation

$$H\psi = E\psi. \tag{1}$$

Here $E \in \mathbb{R}$ and the Hamiltonian H is of the form

$$H = \frac{\hbar^2}{2m}p^2 + V(x), \quad V(x) = -\frac{e^2}{|x|}, \tag{2}$$

where

$$p = \frac{1}{i}\frac{\partial}{\partial x} = \frac{1}{i}\left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right) \tag{3}$$

is the momentum operator conjugate to the position operator $x = (x_1, x_2, x_3)$. With this stationary Schrödinger equation, he could successfully give an explanation of the spectral structure of hydrogen atoms, showing that his formulation of quantum mechanics as the eigenvalue problem of a partial differential operator is valid. Later he proved in [4] that his formulation is equivalent with Heisenberg's formulation of QM. Without loss of generality, we assume $m = 1$ later on.

In the subsequent part [5] he emphasized the necessity to give a time-dependent expression of the equation in order to treat the nonconservative systems, and gave a time-dependent equation for general Hamiltonians

$$\frac{\hbar}{i}\frac{d\psi}{dt}(t) + H\psi(t) = 0. \tag{4}$$

Schrödinger then applied the equation to some time-dependent perturbations with an emphasis of the advantage of the time-dependent approach. He however gave no justification for the notion of time which is assumed for the equation. That is, "time" is postulated [5] to be unique and universally valid throughout the universe as Newton put it in his *Principia Mathematica*.

Exactly the same physical nature of time is assumed for the standard text-book approach to quantum dynamics that is based on the unitary operator $U(t)$, which defines a dynamical map for quantum systems, $\Psi(t) = U(t)\Psi(t=0)$. Hence we can detect the following two assumptions (postulates) built in the fundamental equation for quantum systems dynamics. The first assumption is the equation's mathematical form provided by eq. (4), which here we adopt without modification. The second assumption is that quantum dynamics unfolds within the classical Newtonian universal (global) time. However, at least as a logical possibility, removing the second assumption is not excluded and, if successful, might make the quantum foundations even more efficient—the less number of postulates, the better theory.

Avoiding this assumption is not a trivial task, which we undertake in this paper. Rejecting the in-advance-agreed role of "physical time" for the parameter t in the unitary operator $U(t)$ elevates to the following two related problems. First, if not in advance, then certainly *a posteriori* the role of the parameter t as physical time should be rigorously established; non-rigorous procedures typically

* Corresponding author.
E-mail address: dugic@open.telekom.rs (M. Dugić).

http://dx.doi.org/10.1016/j.physleta.2016.10.010
0375-9601/