# Sharp Cutoff Filters with Monotonic Passband Response


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***Abstract*—Rational functions approximately monotonic in the passband, with one pair of zeros on the imaginary axis, have been compared with allpole functions, monotonic in the passband, for the lowpass filter applications. It is shown that usage of the zeros as a parameter can accomplish a trade between minimum stopband attenuation and sharpness of the cutoff characteristic. The paper concludes with a detailed example showing the efficiency of the proposed technique.**



***Keywords*—Legendre polynomials, Chirstoffel-Darboux identity, orthogonal function, all-pole filter, LC ladder network.**


## I. Introduction

Many types of classical orthogonal polynomials for designing analog low-pass filters have been presented in the literature [1]–[3]. All-pole low-pass filters is an important filter category where transfer functions have all theirs zeros at the infinity. Those functions are easier to implement in comparison to low-pass transfer functions with finite zeros on the imaginary axis, as for example, inverse Chebyshev and Elliptic filters [1]. Thus, all-pole approximations are always considered as a first option for the filter design.

There are approximations that have very good attenuation characteristic at the expense of their phase characteristic, as for example, Chebyshev [1] and Legendre [4], [5]. Opposite case occurs with some other approximations, as for example, Bessel filters [6] which are optimized for maximally-flat constant group delay.

The Legendre orthogonal polynomials have been often used to approximate filter function. The Legendre-Papoulis (known as an "Optimum L" or just "Optimum") filter with monotonic magnitude response in the passband, proposed by A. Papoulis, [5], [7] has the maximum rolloff rate for a given filter degree. It provides a compromise between maximally flat Butterworth filter and Legendre filter [4] with ripples in the pass-band. Legendre-Papoulis filters can be useful in applications that need a steep cutoff at the passband edge but cannot tolerate passband ripples, or in cases where Legendre filter produces very high group delay at the passband edge.

Recently published paper [8] describes the sum-of-squares Legendre polynomial approximation, which offers a smaller deviation of the amplitude characteristic in the passband, but its cutoff slope is equal to cutoff slope of the Optimal filters.

Amplitude and group delay response of these filters are shown in Figure 1.

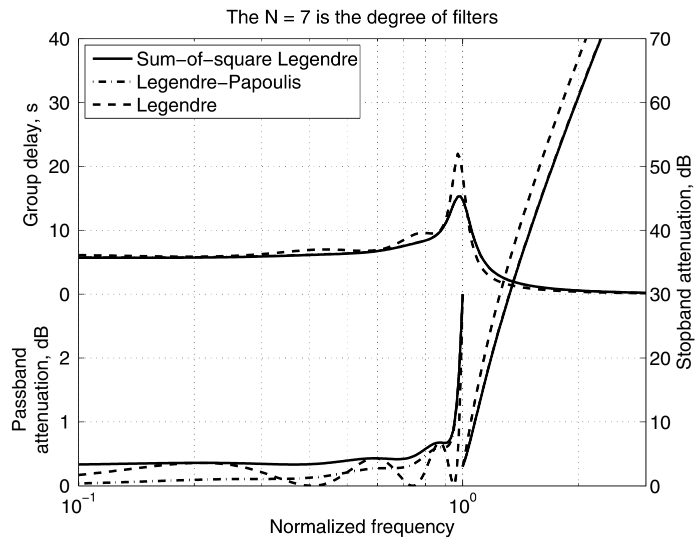


Fig. 1. Amplitude responses and group delay characteristic of the all-pole filters: Legendre, Legendre-Papoulis and sum-of-squares Legendre polynomials

Comparison of the cutoff steepness of filters considered here, can be performed by calculating the slopes of the characteristic function $\Psi(\omega^2)$

$$S = \frac{\mathrm{d}}{\mathrm{d}\,\omega}\Psi(\omega^2)\Big|_{\omega=1} \tag{1}$$

at the cutoff frequency, $\omega_c = 1$, for same attenuation in the passband, $a_{max}$ [9]. $\Psi(\omega^2)$ is the characteristic function given by (3). These slopes are

- Legendre [4]:
$$S_L = n(n+1)$$
- Legender-Papoulis [5], [7]:
$$S_{LP} = \begin{cases} \dfrac{(n+1)^2}{2} & n \text{ is odd} \\ \dfrac{n(n+2)}{2} & n \text{ is even} \end{cases} \tag{2}$$
- Sum-of-squares Legendre polynomials [8]: