Maximally Flat Filter Functions With the Maximum Number of Transmission Zeros Having Maximal Multiplicity

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***Abstract*—The subject of the maximally flat continuous-time filter transfer function with the maximum number of transmission zeros with maximal multiplicity (MFMM) is revisited. It is found that in previous research a simple fact was overlooked: the numerator and denominator polynomials for even-order functions may be of equal orders. New solutions are presented for even-order transfer functions. It is shown that closed-form expressions may be produced for all properties of the amplitude characteristic and the design equations. The function thus obtained is compared with some existing monotonic functions having maximum number, but distinct, transmission zeroes. Starting with a low-pass prototype and using usual transformations, bandpass, bandstop, and high-pass MFMM filter characteristic functions were generated and described for the first time. A design procedure is recommended and design examples are given.**

***Index Terms*—Analog circuits, filtering theory, filters, function approximation, least squares approximations, transfer functions.**

I. Introduction

THE subject of maximally flat low-pass continuous-time filter function with the maximum number of transmission zeros with maximal multiplicity (MFMM) was first discussed in [1] and revisited briefly in [2] and [3]. This brief shows that such amplitude characteristics exhibit a broad stopband region with an extremely high attenuation. That is a unique property of the MFMM functions. In [1]–[3], a restriction is imposed on the number of transmission zeros of the form "$n > 2m$," where n is the filter order and m is the number of finite transmission zeroes on the real-angular-frequency (ω) axis. That restriction is obvious for odd n. For even n, however, there is no justification for that restriction. The filter order is simply allowed to be $n = 2m$.

With this in mind, in this brief we will accomplish three goals. First, to finally complete the proceedings related to the subject of MFMM; second, for the first time, to give closed-form expressions for generation of the MFMM amplitude characteristic for even n; and third, to give new MFMM characteristic functions for bandpass, bandstop, and high-pass cases. Based on these, we will recommend a simple design procedure, too.

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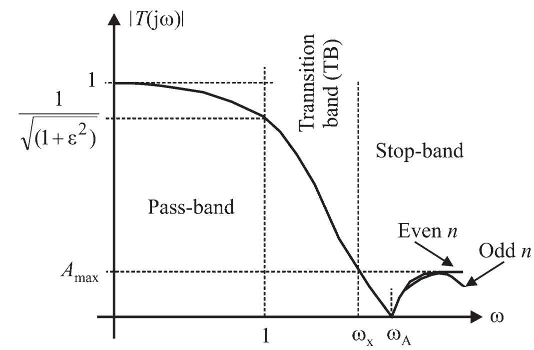

Fig. 1. Definition of the amplitude characteristic parameters.

To get the whole picture, we will put this solution in the context of monotonic passband-amplitude filter design by comparing the MFMM filter with the well-known inverse Chebyshev (IC) [4] (or Chebyshev II) and the least squares monotonic (LSM) [5] filters.

The characteristic function of the MFMM filter may be written as

$$K(\omega^2) = \frac{\left(1-\omega_A^2\right)^{2\cdot[n/2]}}{\left(\omega^2-\omega_A^2\right)^{2\cdot[n/2]}} \cdot \omega^{2n} \tag{1}$$

where [.] denotes the floor function. ω_A denotes the transmission zero (or the attenuation pole) of the filter, as depicted in Fig. 1. Its multiplicity is $[n/2]$. It is assumed in the aforementioned expression that the bandwidth of the filter is normalized to unity.

The first $2n-1$ derivatives of (1) with respect to ω in the origin are equal to zero; hence, it is maximally flat.

The squared amplitude characteristic is given by

$$|T(j\omega)|^2 = 1/\left[1+\varepsilon^2 \cdot K(\omega^2)\right] \tag{2}$$

where ε^2 defines the maximum value of the passband attenuation, which arises at the passband edge, i.e., for $\omega = 1$, where $K(\omega^2) = 1$. From (2), the value of $|T(j\omega)|^2$ for $\omega = 1$ is equal to $1/(1+\varepsilon^2)$, as illustrated in Fig. 1. If the maximum passband attenuation is 3 dB, ε^2 is usually set to unity.

With the maximum passband attenuation, i.e., ε^2, given, there is one more parameter to be defined in order to get the filter function. That is the position of the attenuation pole ω_A. On